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The pessimistic search and the straightening involution for trees. (English)

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The trees T considered here have n vertices labeled 1 (the root of T) through n. Vertex i is said to be above vertex j if vertex i lies in the path joining vertex j to the root. An inversion in T is a pair of vertices (i, j) such that $i < j$ and j is above i. The inversion polynomial for all rooted trees of order n is given by $J_n(q) = \sum_{|T|=n} q^{\mathrm{inv}(T)}$ where $\mathrm{inv}(T)$ is the number of inversions of T. This polynomial was determined by *C. L. Mallows* and *J. Riordan* [Bull. Am. Math. Soc. 74, 92-94 (1968; Zbl 0242.05004)] in terms of a generating function. The pessimistic search on T may be recursively defined as follows. Suppose r is the root of T and $\{T_1, T_2, \ldots, T_k\}$ is the set of subtrees of the root r linearly ordered by the overall minimum elements contained in the subtrees. In the pessimistic search one visits the root first, then recursively searches $T_1, T_2, \ldots, T_k$. The pessimistic order of T is the sequence of vertices of T which reflects the pessimistic search. Next, the straightening involution is developed; it relates the inversion polynomial evaluated at $q = -1$ to the number of even rooted trees. Finally, the author obtains a differential equation for the inversion polynomial of cyclic trees evaluated at $q = -1$ thus solving a problem posed by *I. M. Gessel*, *B. E. Sagan* and *Y.-N. Yeh* [J. Graph Theory 19, No. 4, 435-459 (1995; Zbl 0833.05045)].

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Keywords : inversion polynomial; rooted trees; number of inversions; generating function; pessimistic search

Classification :

∗05A15 Combinatorial enumeration problems
05C30 Enumeration of graphs and maps
05C05 Trees

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