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The combinatorics of a class of representation functions. (English)

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The authors study a class of polynomials that arise from the irreducible representations of the unitary group $U(n)$. These polynomials have been studied extensively in the physics literature where they are formulated in terms of the boson calculus. This paper serves as an introduction for mathematicians and describes what is needed for a rigorous mathematical treatment. They begin this program by proving the following result. Let α and β be compositions of m into n nonnegative integers. Define the homogeneous polynomial of degree m in z_{ij}, $1 \leq i, j \leq n$, by

$$L_{\alpha,\beta}(Z) = \sum_{A \in M(\alpha,\beta)} \prod_{i,j=1}^{n} z_{ij}^{a_{ij}} / a_{ij}!,$$

where $M(\alpha, \beta)$ is the set of $n \times n$ nonnegative integral matrices with row sums given by α and column sums given by β. Let $\alpha!$ be the product of the factorials of the components of α, and let $D_{\alpha,\beta}(Z) = \sqrt{\alpha!\beta!} L_{\alpha,\beta}(Z)$. Let $D(Z)$ be the matrix indexed by the compositions of m with entries $D_{\alpha,\beta}(Z)$. The authors prove that $D(XY) = D(X)D(Y)$ and relate this result to MacMahon's master theorem. They also discuss specializations which include the generalized beta function of Gelfand and Graev, Jacobi polynomials, and Clebsch-Gordon coefficients.

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Keywords : irreducible representations; unitary group; beta function; Jacobi polynomials; Clebsch-Gordon coefficients

Classification :

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20C35 Appl. of group representations to physics

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