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Implication algebras and the Metropolis-Rota axioms for cubic lattices. (English)

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Finite Boolean lattices are exactly the face lattices of n-simplices in n-dimensional Euclidean spaces. Since for $n \geq 5$ all regular solids are either simplices or cubes or hyperoctahedra – the latter two giving rise to dual face lattices – it is an intriguing task to characterize finite cubic (face) lattices internally, too. This task has been solved by *N. Metropolis* and *G.-C. Rota* [SIAM J. Appl. Math. 35, 689- 694 (1978; Zbl 0402.05010)]. Their characterization of finite cubic lattices is given in terms of the mappings Δ_x defined on $[0, x]$, where $\Delta_x(y)$ is the antipodal face of y within the face x. Dropping restrictions of cardinality, the authors of the paper under review show that if L is any not necessarily finite lattice, with 0 and 1, which is equipped with mappings Δ_x for any $x \in L$, satisfying the axioms of Metropolis-Rota, and which furthermore is complete, atomistic, and coatomistic, then it is isomorphic to a possibly infinite cubic lattice (= lattice of pairs of disjoint subsets of some set S). Their proof makes use of the theory of implication algebras [cf. *J. C. Abbott*: Sets, lattices, and Boolean algebras (1969; Zbl 0222.06001)].

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