

LOG CONCAVITY OF THE POWER PARTITION FUNCTION

by

Brennan Benfield

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Mathematics

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Approved By:

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Dr. Arindam Roy

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Dr. William Brian

---

Dr. Evan Houston

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## ABSTRACT

BRENNAN BENFIELD. Log Concavity of the Power Partition Function. (Under the direction of Dr. ARINDAM ROY.)

The main result of this paper is to prove the log concavity of a particular restricted partition  $P_k(n)$  that enumerates the partitions of a positive integer into perfect  $k^{\text{th}}$  powers. Further investigation utilizing MATHEMATICA software yields numerical evidence of certain interesting facts about the function  $P_k(n)$ .

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## ACKNOWLEDGEMENTS

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